IN112: PL formal systems

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1. **A (not so) short proof in $\mathcal{H}$**

   Prove that $p \to p$ is a theorem of the Hilbert formal system $\mathcal{H}$ defined during the lecture.

2. **Gentzen doing yoga**

   Remember the yoga argument:

   > When I am nervous, I practise yoga or relaxation. Someone practising yoga also practises relaxation. So when I do not practise relaxation, I am calm.

   Prove that this argument is correct using $\mathcal{G}$.

3. **Taking the $\mathcal{G}$ train...**

   Using the Gentzen formal system $\mathcal{G}$ for propositional logic defined during the lecture, prove that the following argument is valid:

   > John has travelled by bus or by train. If he has travelled by bus or by car, he has been late and has missed the meeting. He was not late. Therefore he has travelled by train.

4. **Blue eyes**

   John has blue eyes or green eyes and black hair or brown hair. He does not have black hair if he has green eyes. He has blue eyes if he has brown hair.

   Modelize the previous sentences using a propositional language and answer the following questions using Resolution formal system:

   - which color are John's eyes?
   - which color are John's hair?

5. **A simple refutation**

   Prove that the following set of wffs is unsatisfiable:

   $$\{p, p \to q \vee r, q \to r, \neg r, s \vee t\}$$

6. **Taking the $\mathcal{R}$ train...**

   Translate the following formula into Skolem standard form: $((b \vee t) \wedge (b \vee a \to r \wedge m) \wedge \neg r) \to t$. Deduce using the Resolution formal system that the following argument is valid:

   > John has travelled by bus or by train. If he has travelled by bus or by car, he has been late and has missed the meeting. He was not late. Therefore he has travelled by train.

   Is it easier to separate the set of hypotheses from the conclusion to apply the Resolution principle instead of considering the previous formula?

> The following exercises will not be done during lecture. In particular, exercises 8 and 9 are rather theoretical.

7. **Bulb, bulb, bulb**

   (a) prove that the following arguments are valid using the Resolution formal system:

      1. the switch is on and the bulb is not broken if and only if there is light. The bulb is broken. Therefore there is no light.
      2. the switch is on and the bulb is not broken if and only if there is light. There is no light. Therefore either the bulb is broken, either the switch is not on.

(b) there is light if and only if

- the curtains are not closed and it is daylight or
- the switch is on and the bulb is not broken

Show that if the curtains are closed, I have to put the switch on to have light.

8. **Replace all**

   Prove that the replacement theorem introduced in the propositional logic semantics lecture can be expressed through an inference rule in Hilbert system $\mathcal{H}$.

9. **Is $\mathcal{G}$ valid?**

   Prove that the Gentzen formal system $\mathcal{G}$ introduced during the lecture is valid (hint: use structural induction on proof trees).

10. **Are you saturating?**

    Let $\Sigma = \{p \vee q, \neg p \vee q, p \vee \neg q, \neg p \vee \neg q\}$.

    1. prove using the Resolution formal system that Σ is unsatisfaisable using the saturation by level method.
    2. prove using the Resolution formal system that Σ is unsatisfaisable using the saturation by level method and deleting strategy.

11. **Linear deduction**

    Using the Resolution formal system prove that $\{p \vee q, \neg p \vee q, p \vee \neg q, \neg p \vee \neg q\}$ is inconsistant.

    Let S be a set of clauses and C_0 a clause of S. A linear deduction of C_n in S from C_0 is a deduction $(C_0, \ldots, C_n)$ such that:

    - $\forall i \in \{1, \ldots, n\}$, C_i is the resolvent of a clause B_{i-1} and C_{i-1};
    - $\forall i \in \{1, \ldots, n-1\}$, B_i is either a clause of S, either a clause C_j, $j < i$.

    Show that there is a linear deduction of $\square$ in $\{p \vee q, \neg p \vee q, p \vee \neg q, \neg p \vee \neg q\}$.

12. **OL Resolution**

    Prove that $\{a \to b \vee c, e \to d, d \to \neg b, e\} \models \neg(a \wedge \neg c)$ using OL Resolution.