IN112: FOL semantics

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Date :

1. **Natural language sentences modelling**

   Use fist-order logic to model the following declarative sentences. In each case, define precisely the signature of your language.

    1. every rose is a flower
    2. no rose is a flower
    3. some roses are flowers
    4. some roses are not flowers
    5. George is french, John is english and they are friends
    6. all rats and all mice are gray
    7. all giraffes are taller than all rats
    8. cats and dogs are mammals
    9. romans and greeks were enemies
    10. who likes David likes also Tom
    11. David only likes one person
    12. everybody has a father and a mother

2. **Maths, again and again**

   Let E be a set. Model the following mathematical notions using a first-order language. Define precisely the signature of your language.

   (a) $=$ define the "classical" equality relation on E

   (b) $\leq$ is a preorder on E

   (c) $(E, .)$ is a monoid

3. **Skolemization**

   Give a Skolem standard form of the following formulas:

   - $\forall x\ (H(x) \rightarrow ((\exists y\ F(x, y)) \wedge (\exists z\ M(x, z)))$.
   - $(\forall x\ P(x)) \rightarrow (\exists x\ Q(x))$
   - $\forall x \forall y\ (\exists u\ Q(x, y, u) \vee \neg(\exists z\ P(x, z) \wedge P(y, z)))$

The following exercises will not be done during lecture.

4. **Connectors and interpretations**

   Let L_{FOL} be a first-order language, P and Q be two predicate symbols and I be an interpretation. Prove that :

    1. $\models_I \forall x\ P(x)$ iff for all $d \in D_I$ $\langle d \rangle \in \mathcal{I}(P)$
    2. $\models_I \exists x\ P(x)$ iff there is $d \in D_I$ such that $\langle d \rangle \in \mathcal{I}(P)$;
    3. $\models_I \forall x\ P(x) \rightarrow \exists y\ Q(x, y)$ iff for all $d \in D_I$ $\langle d \rangle \in \mathcal{I}(P) \Rightarrow$ there is $d' \in D_I$ $\langle d, d' \rangle \in \mathcal{I}(Q)$.

5. **Socrate**

   Show in model theory and using a first-order language that the following argument is valid:

   - every human is mortal
   - Socrate is human

- therefore Socrate is mortal

6. **Socrate (again)**

   Show in model theory and using a first-order language that the following argument is valid:

   - every human is mortal
   - Socrate is human
   - Socrate is not mortal
   - therefore humans are not mortals

7. **A horse story**

   Show that the following set of sentences is not contradictory:

   - what is rare is expensive
   - a 1€ horse is rare
   - a 1€ horse is not expensive

   Show that if the sentence "there is a 1€ horse" is added to the previous set, then the set is contradictory.